

# Quantum Cosmological Perturbations: Predictions and Observations

Viatcheslav Mukhanov

*Theoretical Physics, Ludwig Maxmillians University, Theresienstr. 37, 80333 Munich, Germany*  
*LPT de l'Ecole Normale Supérieure, Chaire Blaise Pascal, 24 rue Lhomond, 75231 Paris cedex, France*

## Abstract

I consider the generic model independent predictions of the theory of quantum cosmological perturbations. To describe the stage of cosmic inflation, where these perturbations are amplified, the hydrodynamical approach is used. The inflationary stage is completely characterized by the deviation of the equation of state from cosmological constant which is a smooth function of the number of e-folds until the end of inflation. It is shown that in this case the spectral index should deviate from the flat one at least by 3 percent irrespective of any particular scenario. Given the value of the spectral index the lower bound on the amount of the gravitational waves produced is derived. Finally the relation between effective hydrodynamical description of inflation and inflationary scenarios is discussed.

# 1 Introduction

It was discovered in 1980 that the quantum fluctuations of the metric can explain the observable structure of the universe *if and only if the expanding universe* went through a stage of cosmic acceleration [1]. The spectrum of these perturbations in the range of observable scales was calculated for the first time in [2]. About the same time it was realized that in order to understand the large scale homogeneity and isotropy of the observable universe one also needs a stage of accelerated expansion, cosmic inflation [3]. At present there exist hundreds of different inflationary scenarios. To understand what theory of quantum fluctuations really predicts and how to extract the parameters, characterizing the inflationary stage, from observations it is convenient to describe inflation using the effective hydrodynamical approach. In this approach the state of the matter is entirely characterized by its energy density  $\varepsilon$  and the pressure  $p$ . In this note I will only consider the predictive inflationary theory, when both the acceleration and the perturbations are due to the same matter component. There exist many models where one kind of matter is responsible for acceleration and the other for perturbations. In these models nearly any outcome of the measurements can be accommodated making them experimentally non falsifiable and therefore of no great interest. Indeed a theory makes sense only if it makes non-trivial predictions which can be confirmed or disproved by measurements and the best theory is the theory with the minimal number of parameters. In fact, there is no need to involve more parameters unless there appears an obvious contradiction with experimental data or there exist deep theoretical reasons for doing so. Because the theory of simple inflation is in excellent agreement with the present observations it is enough to restrict ourselves to this predictive theory.

## 2 Robust predictions

We assume that in the past the universe went through a stage when matter with equation of state  $p \approx -\varepsilon$  was dominating and, hence, the universe was accelerating. A cosmological constant corresponding to  $p = -\varepsilon$  cannot serve our purpose because finally one has to have a graceful exit from inflation. Therefore, from the very beginning there should be small deviations of the equation of state from the cosmological constant, i.e.,  $(\varepsilon + p)/\varepsilon \ll 1$ , *but nonvanishing*. This ratio, which at the beginning should be small enough to provide us the necessary duration of inflation, grows until it becomes of order unity when inflation ends through a graceful exit to decelerated expansion. One can realize the needed equation of state using the condensates of scalar fields, the  $R^2$  gravity and in some other ways. The key point is that the microscopic origin of the dark energy does not play a crucial role regarding the major predictions of the quantum cosmological perturbations theory. Everything we need is a “decaying cosmological constant”.

To describe how this “cosmological constant” decays we will use as a time

parameter the number of e-fold  $N$  left to the end of inflation, defined as

$$a = a_f \exp(-N), \quad (1)$$

where  $a$  is the scale factor and  $a_f$  is its value at the end of inflation when  $(\varepsilon + p)/\varepsilon \simeq O(1)$ . For observations the relevant interval of  $N$  is not very large, namely,  $N < 70$ . Moreover, the spectrum of fluctuations observed in CMB corresponds to even smaller interval,  $N \simeq 50-60$ . Making the reasonable assumption that during this rather short range of  $N$  the change of the equation of state is monotonic and smooth and taking into account that  $(\varepsilon + p)/\varepsilon \simeq O(1)$  at  $N = 0$ , it is rather natural to approximate the equation of state by

$$1 + \frac{p}{\varepsilon} = \frac{\beta}{(N + 1)^\alpha}, \quad (2)$$

where  $\alpha$  and  $\beta$  are both positive and of order unity. Within this set up, irrespective of the initial conditions for the perturbations<sup>1</sup>, concrete robust predictions for observations can be derived. What are these predictions?

First of all if inflation last more than 70 e-folds the cosmological parameter  $\Omega$  should be equal to unity within an accuracy about  $10^{-5}$ , which means that at present the universe has a flat Euclidean geometry. This prediction was first confirmed only at the end of 90th with the discovery of dark energy. Note that the experimental data before were in strong disagreement with it.

The other set of the robust predictions concerns the amplified quantum fluctuations [2]. More concretely:

-The produced inhomogeneities should be adiabatic. I would like to stress that about thirty years ago the observational data were more supportive for entropy perturbations. However, nowadays they are ruled out by the precision CMB measurements, which confirmed the adiabatic nature of the primordial inhomogeneities.

-The primordial inhomogeneities are nearly Gaussian. This is because they were originated as the result of amplification by the external classical gravitational field of the initial gaussian fluctuations. The expected corrections to the gaussian gravitational potential  $\Phi_g$ , due to nonlinear corrections to the linearized Einstein equations are of order  $O(1)\Phi_g^2$ , that is,  $\Phi = \Phi_g + f_{NL}\Phi_g^2$ . The present experimental bound  $-10 < f_{NL} < 70$  is in agreement with the prediction of the theory, according to which  $f_{NL}$  is expected to be about ten. The forecasted accuracy of the Planck mission  $\Delta f_{NL} \simeq 5$  will allow us to improve further the measurements of the non-gaussianity.

-The most nontrivial prediction for the perturbations is a weak scale dependence of the amplitude of the gravitational potential  $\Phi$ . Namely, the amplitude of  $\Phi$  must logarithmically depend on the scale  $\lambda$  and grow towards the larger

---

<sup>1</sup>It is often mistakenly stated in the literature that one has to postulate the initial "Bunch-Davies vacuum" for the cosmological perturbations. In fact assuming that duration of inflation lasts longer than 70 e-folds the spectrum generated in the observable scales does not depend on the initial conditions for the perturbations provided that they do not destroy via backreaction the inflationary stage from the very beginning.

scales. Within the observable range of scales the logarithm can be approximated by  $\Phi^2 \propto \lambda^{1-n_s}$ . The rather generic formula for the spectral index  $n_s$  is [4]:

$$n_s - 1 = -3 \left(1 + \frac{p}{\varepsilon}\right) + \frac{d}{dN} \ln \left(1 + \frac{p}{\varepsilon}\right). \quad (3)$$

For a monotonic change of the equation of state both terms are negative and hence inflation *always* predicts the red-tilted spectrum. Substituting (2) into (3) we find that

$$n_s - 1 = -\frac{3\beta}{(N+1)^\alpha} - \frac{\alpha}{(N+1)} \quad (4)$$

For  $\alpha < 1$  the first term on the r.h.s. dominates, while for  $\alpha > 1$  the second term is more important. In case of  $\alpha = 1$  both terms give a comparable contribution. Considering  $n_s - 1$  as a function of  $\alpha$  we find that the minimal deviation from the flat spectrum ( $n_s = 1$ ) corresponds to

$$\alpha = 1 + \frac{\ln(3\beta \ln(N+1))}{\ln(N+1)}. \quad (5)$$

Substituting this expression in (4) we will find that inflation predicts that the deviation of the spectral index from unity must be larger than

$$n_s - 1 = -\left(1 + \frac{\ln(3\beta e \ln(N+1))}{\ln(N+1)}\right) \frac{1}{(N+1)}. \quad (6)$$

Taking  $3\beta = 1$  ( $\beta$  cannot be much smaller than unity) and noting that  $N = 50$  for the scales where this spectral index is measured, we find that the predicted spectral index should be smaller than  $n_s = 0.968$ . The logarithmic spectrum obtained in [2] corresponds to  $n_s = 0.96$  and this prediction is in good agreement with the most recent measurements of the CMB fluctuations [5], which give  $n_s = 0.9690 \pm 0.0089$  and confirm the logarithmic dependence of the gravitational potential at the level of  $3.5\sigma$ . This logarithmic dependence has a deep physical origin since it is due to the small deviation of the equation of state from cosmological constant needed for a graceful exit.

If any of the above predictions would contradict to the observations then inflation as a predictive theory (sometimes called simple inflation) would be ruled out.

-One more robust prediction of inflation is the existence of the longwave gravitational waves [6]. The ratio of the tensor ( $T$ ) to scalar mode ( $S$ ) can also be expressed in terms of the equation of state (see [4]):

$$r = \frac{T}{S} = 24 \left(1 + \frac{p}{\varepsilon}\right) = \frac{24\beta}{(N+1)^\alpha} \quad (7)$$

One can immediately see that taking, for instance,  $\alpha = 2$  we can reduce the amount of the longwave gravitational waves by a factor  $N = 50$  compared to the case  $\alpha = 1$ . Hence, the particular value of the ratio  $r$  is not predicted by generic theory of inflation. However, after spectral index  $n_s$  is measured one can

establish most likely lower bound on the amount of the generated gravitational waves. This bound does not depend on the particular inflationary scenario. In fact, for  $\alpha > 1$ ,

$$n_s - 1 \simeq -\frac{\alpha}{N}, \quad (8)$$

and if, for instance, the measured value  $n_s \simeq 0.96$ , then  $\alpha$  cannot be larger than 2 and, hence, the lower bound on  $r$  is

$$r = \frac{24\beta}{(N+1)^2} \simeq 10^{-2}\beta, \quad (9)$$

where  $\beta$  is of order unity. For the spectral index  $n_s \simeq 0.94$  the amount of the gravity waves can be further suppressed by a factor 50. Hence, the non-detection of the gravitational waves in the current CMB measurements does not rule out the predictive inflationary theory, but on the other hand their detection would provide an extra strong evidence for inflation.

I would like to stress that the model independent predictions above are extremely nontrivial and were for a long time in conflict with observations. For example, in the 80th, along with the theory of quantum initial perturbations there were competing theories of cosmic strings, textures and entropy perturbations, which sometimes were even more favorable from the point of view of observations. However, now all these theories are ruled out and only the theory of quantum cosmological perturbations with all its nontrivial predictions is confirmed by observations. Moreover, although there are still claims in the literature that there are alternatives to inflation, there is no any alternative to the quantum origin of the universe structure.

After the origin of the universe structure from quantum fluctuations is confirmed one can ask the question how much can we really learn about fundamental physics making precise measurements of the parameters  $\alpha$  and  $\beta$ .

### 3 Slow roll inflation

Assuming that inflation is due to the slow roll scalar field with a standard kinetic energy term we will determine the scalar field potentials, which correspond to different values of  $\alpha$  and  $\beta$ . Keeping in mind that the energy density during inflation is approximately equal to the potential, that is,  $\varepsilon \simeq V(\varphi)$ , we first determine how the energy density depends on  $N$ . The energy conservation equation

$$\dot{\varepsilon} = -3H(\varepsilon + p), \quad (10)$$

where  $H = \dot{a}/a$  and the dot denotes the time derivative, can be rewritten as

$$\frac{d \ln \varepsilon}{dN} = 3 \left(1 + \frac{p}{\varepsilon}\right) = \frac{3\beta}{(N+1)^\alpha}. \quad (11)$$

Integrating this equation we obtain

$$\varepsilon(N) \simeq \begin{cases} \varepsilon_f (N+1)^{3\beta}, & \alpha = 1, \\ \varepsilon_0 \exp\left(-\frac{3\beta}{\alpha-1} \frac{1}{(N+1)^{\alpha-1}}\right), & \alpha \neq 1. \end{cases} \quad (12)$$

To determine the slow roll potential  $V(\varphi) \simeq \varepsilon$  we have to express  $N$  in terms of the scalar field  $\varphi$ . With this purpose we write

$$\frac{d\varphi}{dN} = \frac{\dot{\varphi}}{-H} = \sqrt{\frac{3}{8\pi} \left(1 + \frac{p}{\varepsilon}\right)}, \quad (13)$$

where we have taken into account that  $\varepsilon + p = \dot{\varphi}^2$  and  $H^2 = 8\pi\varepsilon/3$  (in the Planck units). Substituting here (2) and integrating the resulting equation we find

$$N+1 = \begin{cases} \exp\left(\pm\sqrt{\frac{8\pi}{3\beta}}(\varphi+C)\right), & \alpha = 2, \\ \left[\frac{2\pi}{3\beta}(2-\alpha)\right]^{\frac{1}{2-\alpha}} (\pm\varphi+C)^{\frac{2}{2-\alpha}}, & \alpha \neq 2, \end{cases} \quad (14)$$

where  $C$  is a constant of integration.

Let us consider the cases  $\alpha = 1$ ,  $\alpha = 2$  and  $\alpha \neq 1, 2$  separately.

-  $\alpha = 1$

Taking into account that during slow roll  $V(\varphi) \simeq \varepsilon$  and combining (12) and (14) we find that

$$V(\varphi) \simeq V(\varphi_f) \left(\frac{\varphi}{\varphi_f}\right)^{6\beta}, \quad (15)$$

where  $\varphi_f = (3\beta/2\pi)^{1/2}$ . Thus, the case of  $\alpha = 1$  corresponds to chaotic inflationary scenarios with the power law potentials[7]. In this case both terms in (3) give comparable contributions to spectral index, which becomes

$$n_s - 1 = -\frac{3\beta + 1}{N + 1}, \quad (16)$$

and the amount of the gravitational waves produced is rather substantial

$$r = \frac{24\beta}{N + 1}. \quad (17)$$

For instance, for the massive scalar field ( $3\beta = 1$ ) we have  $n_s \simeq 0.96$  and  $r \simeq 0.16$  for  $N = 50$ , while in the case  $3\beta = 2$ , that is, for quartic potential  $n_s \simeq 0.94$  and  $r \simeq 0.32$ . The measured value of the spectral index  $n_s = 0.9690 \pm 0.0089$  and the upper bound on the amount of the gravitational waves seems disfavor the quartic potential at rather high significance level. The model of massive scalar field predicts the spectral index to be in agreement with observations, but the amount of the produced gravitational waves seems too high although one cannot yet definitely rule out this model [8].

-  $\alpha = 2$

In this case the second term in (3) dominates. The spectral index does not significantly depends on  $\beta$  and is equal to

$$n_s - 1 \simeq -\frac{2}{N}. \quad (18)$$

For  $N = 50$  the spectral index  $n_s \simeq 0.96$  and the amount of the gravitational waves  $r \simeq 10^{-2}\beta$  are both in very agreement with observations. As it follows from (12) and (14) this case corresponds to the following slow roll potential

$$V(\varphi) \simeq V_0 \exp \left( -3\beta \exp \left( \mp \sqrt{\frac{8\pi}{3\beta}} (\varphi + C) \right) \right), \quad (19)$$

Taking properly the constant of integration  $C$  and expanding this potential for large  $\varphi$

$$\begin{aligned} V(\varphi) &\simeq V_0 \left[ \exp \left( -\frac{3\beta}{2} \exp \left( -\sqrt{\frac{8\pi}{3\beta}} (\varphi + C) \right) \right) \right]^2 \\ &\simeq V_0 \left[ 1 - \exp \left( -\sqrt{\frac{8\pi}{3\beta}} \varphi \right) \right]^2, \end{aligned} \quad (20)$$

we immediately recognize, for  $\beta = 1/2$ , the  $R^2$ -inflation [9] and Higgs inflation [10], which are indistinguishable experimentally. They predict not only the same spectral index but also exactly the same amount of the gravitational waves  $r = 12/(N+1)^2 \simeq 5 \times 10^{-3}$ . The other models in this class correspond to different values of  $\beta$  and can be distinguished only by the amount of the produced gravitational waves.

$\alpha \neq 1, 2$

In this case the potential is

$$V(\varphi) \simeq V_0 \exp \left[ -\frac{3\beta}{\alpha-1} \left( \frac{2\pi}{3\beta} (2-\alpha) \varphi^2 \right)^{\frac{\alpha-1}{\alpha-2}} \right]. \quad (21)$$

First of all note that if  $\alpha = 0$  then  $1 + p/\varepsilon = \beta$  and for  $\beta \ll 1$  we have inflation which continues forever without graceful exit. As one can see from (21) this case corresponds to the exponential potential  $V(\varphi) \simeq V_0 \exp(\sqrt{12\pi\beta}\varphi)$ . To understand the difference between the cases  $0 < \alpha < 1$ ,  $1 < \alpha < 2$  and  $\alpha > 2$  we will consider separately  $\alpha = 1/2, 3/2$  and  $\alpha = 3$ . As one can see from (14) the first two cases correspond, as before, to large field inflation, while in the third case we have small field inflation.

For  $\alpha = 1/2$  the potential becomes

$$V(\varphi) \simeq V_0 \exp \left( 6\pi^{1/3} (\beta\varphi)^{2/3} \right) \quad (22)$$

and for  $\varphi > 1$  it satisfies the slow roll conditions. For instance, in case when  $3\beta = 1$ , the spectral index  $n_s \simeq 0.85$  and  $r \simeq 1.13$  are in obvious contradiction with observations.

The case  $\alpha = 3/2$  corresponds to the potential

$$V(\varphi) \simeq V_0 \exp\left(-\frac{27\beta}{\pi\varphi^2}\right) \simeq V_0 \left(1 - \frac{27\beta}{\pi\varphi^2}\right), \quad (23)$$

for  $\varphi > 1$ , where this potential is trustable. The situation here is similar to  $\alpha = 2$  case, where for large  $\varphi$  the potential approaches a constant value exponentially fast. Here this constant is approached only as an inverse power law. The spectral index  $n_s \simeq 0.967$  and  $r \simeq 2 \times 10^{-2}$  in good agreement with observations. It seems that taking into account the possible accuracy of the measurements, the uncertainty in  $N$  due to unknown detailed physics after inflation and the remaining freedom in the choice of  $\beta$  one will never be able to distinguish the models described by potentials of type (19) and (23) even if one will ever measure the gravitational waves. On the other hand, the models with potentials as in (15) and (22) seems could soon be ruled out even if one only improve the accuracy of the determination of the spectral index and the upper bound on  $r$ , for instance, in Planck experiment. Therefore, although non-decisive for selecting a particular scenario the precision measurements are very useful for excluding the whole families of inflationary scenarios.

Finally let us consider the case of  $\alpha = 3$ ,

$$V(\varphi) \simeq V_0 \exp\left(-\frac{2\pi^2}{3\beta}\varphi^4\right) \simeq V_0 \left(1 - \frac{2\pi^2}{3\beta}\varphi^4\right) \quad (24)$$

Here inflation occurs at small values of scalar field. The spectral index corresponds to  $n_s \simeq 0.94$  and the amount of the gravity waves is further suppressed by  $N$  compared to  $\alpha = 2$  case, namely,  $r = 24\beta/(N+1)^3 \simeq 6 \times 10^{-5}$  for  $3\beta = 1$ .

The potentials above approximate the inflationary potential only in the range of slow roll regime, or even more conservatively within last 70 e-folds of inflation. It is clear that outside this range they can be changed rather arbitrarily.

## 4 k-Inflation

Until now we have characterized inflation only by one function, namely, by  $N$ -dependence of deviation of the equation of state from cosmological constant. However, yet remaining in one fluid approximation one can introduce the other natural parameter which describes this fluid, namely, the speed of propagation of cosmological perturbations, or in other words, the speed of sound  $c_s$ , which naturally arises in k-inflation [11]. At present to explain the observations there is no need to assume that  $c_s$  is different from unity. However, because slow roll and k-inflation scenarios belong to the same class of simplicity it is interesting to check how this extra parameter will influence the robust predictions above. In case of  $c_s \neq 1$  the formulae (3) and (7) are modified as [4]:

$$n_s - 1 = -3 \left(1 + \frac{p}{\varepsilon}\right) + \frac{d}{dN} \ln \left[c_s \left(1 + \frac{p}{\varepsilon}\right)\right], \quad (25)$$



and

$$r = \frac{T}{S} = 24c_s \left(1 + \frac{p}{\varepsilon}\right). \quad (26)$$

We can parametrize k-inflation with

$$1 + \frac{p}{\varepsilon} = \frac{\beta}{(N+1)^\alpha}, \quad c_s = \frac{\gamma}{(N+1)^\delta}, \quad (27)$$

where  $\delta \geq 0$  because the speed of sound generically grows towards the end of inflation[11], and  $\gamma$  is now an arbitrary positive number not necessarily of order unity. The expression for the spectral index becomes

$$n_s - 1 = -\frac{3\beta}{(N+1)^\alpha} - \frac{\alpha + \delta}{(N+1)}, \quad (28)$$

and because  $\delta$  is positive the conclusion about minimal deviations of the spectral index from unity remains unaltered. On the other hand it looks like the tensor to scalar ratio

$$r = \frac{24\beta\gamma}{(N+1)^{\alpha+\delta}}, \quad (29)$$

can be made arbitrarily small because  $\gamma$  is not needed to be of order unity and as a result the lower bound on the amount of gravitational waves will disappear. However, it is not so. In fact, too small speed of sound induces too large primordial non-gaussianities of order  $f_{NL} \simeq O(1)/c_s^2$ . Therefore taking the experimental bound on  $f_{NL} < 80$  [8], we conclude that the speed of sound cannot be much smaller than 0.1 and hence for the spectral index  $n_s \simeq 0.96$  the lower bound on  $r$  (see (9)) is not modified more than by an order of magnitude at maximum.

## 5 Discussion

We have shown that the majority of inflationary scenarios can be parametrized by two numbers and using this we were able to prove the bound on the minimal deviations of spectral index from unity and to obtain the lower bound on the amount of the gravitational waves produced. Although primordial gravitational waves are not yet detected, the experimental confirmation of the flatness of the universe, adiabatic nature of nearly gaussian perturbations and the discovered (at 3,5 sigma level) logarithmic tilt of the spectrum unambiguously prove the quantum origin of the universe structure and the early cosmic acceleration. Needless to say that all these predictions, which were yet in conflict with observations about 15 years ago, are very nontrivial. Given that the quantum origin of the universe structure is experimentally confirmed, the precision measurements already now allow us to exclude many inflationary scenarios existing in the literature. Moreover, the improved accuracy of the determination of spectral index, the bound (or detection) on non-gaussianity and the bound (or possible future detection) on primordial gravitational waves will allow us to put further

restrictions on the admissible inflationary scenarios. However, this seems will not help us too much in recovering the fundamental particle physics behind inflation. In fact, the observational data only allow us to measure only the effective equation of state and the rate of its change in a rather small interval of scales. Keeping in mind unavoidable experimental uncertainty, the effect of unknown physics right after inflation and degeneracy in the scenarios discussed above we perhaps will never be able to find out the microscopical theory of inflation without further very essential input from the particle physics. On the other hand, the remarkable property of the theory of quantum origin of the universe structure is that the gravity seems does not care too much about microscopic theory providing needed equation of state, and allows us to make experimentally verifiable predictions.

**Acknowledgements** I am grateful to Cesar Gomez for useful discussions. This work was supported by “Chaire Internationale de Recherche Blaise Pascal financée par l’Etat et la Région d’Île-de-France, gérée par la Fondation de l’Ecole Normale Supérieure”, by TRR 33 “The Dark Universe” and the Cluster of Excellence EXC 153 “Origin and Structure of the Universe”.

## References

- [1] G. V. Chibisov and V. F. Mukhanov, *Galaxy Formation and Phonons*, Lebedev Phys. Inst. Preprint No. 162, 1980. This paper was submitted for publication to MNRAS in October 1980 but because of inefficiency of the editorial office was published with two years delay in MNRAS vol. **200**, pp. 535-550, 1982.
- [2] V. F. Mukhanov and G. V. Chibisov, *Quantum fluctuation and nonsingular Universe*, JETP Letters, vol. **33**, pp. 532–535, 1981.
- [3] A. H. Guth, *Inflationary universe: a possible solution to the horizon and flatness problems*, Physical Review D, vol. **23**, no. 2, pp. 347–356, 1981; A. D. Linde, *A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems*, Physics Letters B, vol. **108**, no. 6, pp. 389–393, 1982; see also: R. Brout, F. Englert, and E. Gunzig, *The Causal Universe*, General Relativity and Gravitation, vol. **10**, no. 1, pp. 1-6, 1979.
- [4] V. Mukhanov, *Physical Foundations of Cosmology*, Cambridge University Press, Cambridge, UK, 2005.
- [5] E. Calabrese et.al., *Cosmological Parameters from Pre-Planck CMB Measurements*, e-Print: arXiv:1302.1841 [astro-ph.CO], 2013.
- [6] A. Starobinsky, *Spectrum of Relict Gravitational Radiation and the Early State of the Universe*, JETP Lett. vol. **30**, pp. 682-685, 1979.
- [7] A. Linde, *Chaotic Inflation*, Phys. Lett. B, vol. **129**, pp. 177-180, 1983.

- [8] G. Hinshaw et. al., *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results*, e-Print: arXiv:1212.5226 [astro-ph.CO], 2012.
- [9] A. Starobinsky, *A New Type of Isotropic Cosmological Model without Singularity*, Phys. Lett. B, vol. **91**, p. 99, 1980.
- [10] F. L. Bezrukov and M. E. Shaposhnikov, *The Standard Model Higgs Boson as the Inflaton*, Phys. Lett. B, vol. **659**, pp. 703-706, 2008
- [11] C. Armendáriz-Picón, T. Damour, and V. Mukhanov, *k-inflation*, Physics Letters B, vol. **458**, no. 2-3, pp. 209–218, 1999.